"Dynamic Response of a Beam Structure to a Moving Mass Using Green's Function"

A THESIS SUBMITTED IN PARTIAL FULFILLMENT
OF THE REQUIREMENTS FOR THE AWARD OF

Master of Technology
In
Machine Design and Analysis

By

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Department of Mechanical Engineering

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Under the Guidance of

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ROURKELA

CERTIFICATE

This is to certify that the thesis entitled, "**Dynamic Response of a Beam Structure to a Moving Mass Using Green's Function**" by **Sudhansu Meher** in partial fulfillment of the requirements for the award of **Master of Technology** Degree in **Mechanical Engineering** with specialization in "**Machine Design & Analysis**" at the National Institute of Technology, Rourkela is an authentic work carried out by him under my supervision and guidance.

To the best of my knowledge the matter embodied in the thesis has not been submitted to any other University/ Institute for the award of any Degree or Diploma.

Date**:**

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CONTENTS

ABSTRACT

The dynamic responses of a beam subjected to a moving load or moving masses have been of importance in the design of railway tracks and bridges and machining processes. The importance of this problem find uses in many applications in the field of transportation, Bridges, guide ways, overhead cranes, cableways, rails, roadways, runways, tunnels and pipelines. These structural elements are designed to support moving masses. The inertial effect of the moving mass cannot be ignored in comparison with the gravitational effect even if the velocity of the moving mass is comparatively little. In the present study the equation of motion in matrix form for an Euler beam subjected to a concentrated mass moving at a steady speed is formulated by using the Green's function approach. The solutions of the declared problems for the case of simply supported and cantilever beams are evaluated by using Dynamic Green's function. In the present work, the effect of the moving mass and its speed on the dynamic response of simply supported and cantilever beam for two different materials have been investigated. An experiment is also conducted which supports the analytical results. The beam is divided into twenty divisions and the deflection of beam at mid-span is recorded while a mass moves at constant velocity over the beam through different stations. The deflection at mid-span is recorded when the mass is at different stations with the help of a oscilloscope. The result obtained is plotted in the form of graphs for different velocities of mass.

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NOMENCLATURE

E = Young's Modulus of the Respective Material

I = Second Moment of Area of the Beam Cross-Section

m = Mass per unit length

x = Axial co-ordinate

t = Time

w (x,t) = The transverse deflection of the beam

F = Applied force

u = Position of the mass on the beam

δ (x-u) = Dirac Delta function

M = Weight of the moving mass

g = Gravitational acceleration

β = Transverse displacement of the mass

G (x,u) = Dynamic green function

q = Frequency parameter

ω = Circular frequency

v = Speed of the moving mass

L = Length of the beam

A_1 to A_4, B_1 to B_4 = Constants in the green's function equation

h = Interval length

N = Number of stations

i = Any one of the N discrete station points

a_k= Coefficient in the derivative approximating equation

[I] = Identity matrix

{ Δ_{uj}} = A column matrix which is having unity at 'j' th row

CHAPTER 1

-INTRODUCTION-

1. INTRODUCTION

The moving load problem is a fundamental problem in structural dynamics. Engineers have been investing the potential hazard produced by the moving masses on structures for many several years. The dynamic response of structures carrying moving masses is a problem of widespread practical significance. A lot of hard work has been accounted during the last ten decades relating with the dynamic response of railway bridges and later on highway bridges under the effect of moving loads. Beam type structures are widely used in many branches of civil, mechanical and aerospace engineering. The importance of moving mass is found in several applications in the field of transportation. Railway and highway bridges, suspension bridges, guide ways, crane runways, cableways, rails, roadways, runways, tunnels and pipelines are example of structural elements to be designed to support moving masses. Also, in the design of machining processes, many members can be modeled as beams acted upon by moving loads.

The dynamic effect of moving loads was not known until mid-nineteenth century. When the Stephenson's bridge across river Dee Chester in England in 1947 collapsed, it motivates the engineers for research of moving load problem. Moving loads have a great effect on the bodies or structures over which it travels. It causes them to vibrate intensively, especially at high velocities. The peculiar features of moving loads are they are variable in both space and time. Modern means of transport are ever faster and heavier, while the structures over which they move are ever more slender and lighter. That is why the dynamic stresses they produce are larger by far than the static ones.

The majority of the engineering structures are subjected to time and space varying loads. Moving loads have substantial effects on the dynamic behavior of the engineering structures. Increase in traffic intensity and speed requires more multifarious study of structures than it was case before. The simplest case of a moving load investigation is the case of a simple beam over which a concentrated load is moving, that is represented with a Fourth order partial differential equation

This problem has significant effect in civil and mechanical engineering. The dynamic analysis of the vibrating beam is done by neglecting the disconnection of the moving mass from the beam during the motion and result is given by considering mass moving at constant speed and in one direction. Once the load departs from the beam, it begins to vibrate at in free vibration mode. Hence this process no longer comes within the scope of the experiment.

CHAPTER 2

-LITERATURE REVIEW-

2. LITERATURE REVIEW

The dynamic analysis of beam structure with moving load is a fundamental problem in structural dynamics. In comparison to other dynamic load, the moving load varies in position as well as time and that's why the moving load problem is a special topic in structural dynamics. Since nineteenth century the moving load problem has become more dynamic in nature due to increased vehicle speed and structural flexibility. The problem of dynamic response of Bernoulli-Euler beam subjected to a moving mass has been studied by many authors and the importance of this problem is demonstrated in several ways by many authors.

The response of beams under the action of moving loads has attracted the attention of many scientists since the nineteenth century. Till today, various kinds of problems associated with moving loads have been studied, and many explicit solutions were found. Various kinds of problems associated with moving loads have been presented in the book by Fryba [1]. A lot of work has been reported during the past hundred years dealing with the dynamic response of railway bridges and highway bridges under the action of moving loads. Extensive references to the literature on the subject can be found in the book by Fryba[1]. The dynamic behavior of beam structures, such as bridges on railways, subjected to moving loads or masses has been investigated for over a century. There are numerous reports available in the book by Fryba [1], and most of them treat a uniform simply supported beam of single span. Fryba [1] presented a comprehensive study of the methods proposed for solving the problem; numerical results obtained by using infinite series were also presented in his book. Prior to his work, a lot of studies were made to the problem of the vibration of simple and continuous beams under moving loads. These results are available in the textbook. He used the Fourier sine integral transformation and the Laplace–Carson integral transformation to determine the dynamic response of beams due to moving loads and obtained this response in the form of series solutions.

Mackertich [2] used the modal superposition method for the beam deflection and compared the response of a Timoshenko beam to a moving mass to that of a Bernoulli-Euler one. He approximated the total time derivative of the mass displacement by the partial derivative to put off the difficulty that arises from the existence of a coupling term in the mass acceleration expression. The problem of dynamic response of a Bernoulli-Euler beam subjected to a moving mass has been studied by many others. However response of a beam to a moving mass with corrections for shear deformation and rotary inertia effects, which may be important for high speeds of moving mass, has not received much attention. The solution is based on the beam theory with correction for the shear deformation and rotary inertia. The effect of shear deformation and rotary inertia are significant in determining the dynamic response of a beam excited by a high-velocity moving mass.

Hamada [3] used double Laplace transformation method to find a solution for a beam with damping under the action of a moving mass less load. Hamada solved the response problem of a simply supported and damped Bernoulli-Euler uniform beam of finite length traversed by a constant force moving at a uniform speed by applying the double Laplace transformation with respect to both time and the length coordinate along the beam. He obtained an exact solution in closed form for the dynamic deflection of the considered beam. A method has been presented, based on the double Laplace transformation, for dynamic analysis of a simply supported and damped Bernoulli-Euler uniform beam of finite length subjected to the action of a moving concentrated force. The transformed formula thus obtained has been applied to the case in which a constant force is moving at a uniform speed, and the solution of the problem has been found out.

Michaltsos et al [4] follows the first approximation technique to derive a series solution for beam dynamic deflection in terms of beam normal modes without the effect of the mass inertia. He studied the linear dynamic response of a simply supported uniform beam under a moving load of constant magnitude and velocity by including the effect of its mass. Using a series solution for the dynamic defection in terms of normal modes the individual and coupling effect of the mass of the moving load, its velocity and of other parameters are fully assessed. A variety of numerical results allows us to draw important conclusions for structural design purposes. The individual and coupling effects, on the dynamic response of various parameters such as load mass and velocity of the moving load, are discussed in detail. In this case the effect of the load mass on the dynamic response of the beam is neglected.

Ting et al [5] formulated and solved the problem using the influence coefficients method. The inertial effects of the beam were considered as applied external forces, and again at each position of the mass, numerical integration had to be performed over the length of the beam. He developed an algorithm to solve the classical problem of the dynamic response of a finite elastic beam supporting a moving mass. The results are very well comparable with the experimental results. He has presented a general algorithm for studying the response of both the mass and the beam for moving mass problems. The boundary conditions are imbedded in the algorithm and hence only initial conditions are required to have a well defined problem. He used homogeneous initial conditions to properly focus on the boundary value problem studied by others.

C. W. Bert [6] presented comparative evaluation of six different numerical integration methods. He examined linear and non-linear, undamped and damped systems. In some cases, the exact solutions are known; this provides a concrete basis for comparing the accuracy of the results. However, in other cases, a converged solution is used as a basis of comparison. The converged solution is obtained by continuing to solve the problem for smaller and smaller time steps until the response converges to a fixed-time curve. The methods are studied for stability and computational efficiency. Stability is measured by the performance of the integration scheme as the solution time step is gradually increased. Numerical efficiency is gauged by the total time required to calculate the system response. The motivation for this analysis is to select a suitable integration scheme for solving non-linear transient problems containing higher degrees of freedom.

A. S. Mohamed [7] deals with the construction and tabulation of Green's functions that is suitable for determining natural frequencies and mode shapes of beams with intermediate attachments and of various boundary conditions. The beam may have rotational and linear elastic attachments, also rotational and linear attached inertias. An exact method for determining the dynamic characteristics of Euler-Bernoulli beams with attached masses and springs is given, using Green's functions. These functions have been tabulated for beams of several boundary conditions. Some problems with known solutions are considered, and the results confirm the correctness of the method.

H. P. Lee [8] studied extensively the dynamic responses of a beam acted upon by moving forces or moving masses, in connection with the design of railway tracks and bridges and machining processes. The equation of motion in matrix form has been formulated for the dynamic response of a beam acted upon by a moving mass by using the Lagrangian approach. Convergence of numerical results is found to be achieved with just a few terms for the assumed deflection function. The present numerical results in dimensionless form enable the results to be applicable for a large combination of system parameters. It is found that separation of the mass from the beam may occur for a relatively slow speed and small mass when the beam is clamped at both ends.

M. Ichikawa et al [9] investigated the behavior of the multi-span continuous beam acted upon by a moving mass at a constant velocity, in which it is assumed that each span of the continuous beam obeys uniform Euler-Bernoulli beam theory. The solution to this system is simply obtained by taking both Eigen function expansion or the modal analysis method and the direct integration method combined. The effects of the inertia and the moving velocity of the load on the dynamic response of the continuous beam are solved for three kinds of continuous beams having uniform span length. The behavior of a multi-span continuous beam acted upon by a moving mass with a constant velocity has been studied. The method used for solving the present problem is the Eigen function expansion or modal analysis accompanied by the direct integration method, and it can also easily include other effects such as non-uniformity of the continuous beam, various combinations of boundary conditions and the speed variation of the moving mass. Numerical calculations have been done to clarify the effects of two important parameters, the mass ratio of the moving mass to the first span and the velocity of the moving mass, on the dynamic response and the amplification factor of the continuous beams having uniform span length.

J. D. Achenbach et al [10] obtained the solutions that are time invariant in a coordinate system moving with the load velocity. The supporting foundation has damping effects. The effect of the damping coefficient and the load velocity on the beam response is studied. The limiting case of no damping is included and the various resonance effects are studied. It has been shown that transverse foundation damping decreases the magnitudes of the displacement and the bending moment. The discontinuities under the load in the slope of the displacement and the second derivative of the rotation are not affected by transverse foundation damping.

G.G.G. Lueschen et al [11] studied the closed form solution for the Green's functions of individual elements for the analysis of the systems. In his paper, a concise formulation is given for the Green's functions of uniform Timoshenko beams. It is shown that Green's functions for uniform Euler-Bernoulli beams, both with and without constant axial loads, can be expressed in the same form. Green's functions have been derived for Timoshenko beams, as well as for Euler-Bernoulli beams with and without a constant axial load, in closed form showing the relationships between the two existing theories. The compactness of the results and the fact that each is simply expressed over the entire domain make these forms especially suitable for analysis and computation.

M. Mofid et al [12] extended a procedure that has been used to discretize a continuous flexible beam into a system of rigid bars and joints, which resist relative rotation of the attached bars. The object of this article is to present and formulate a simple and practical approximate technique for determining the response of beams with internal hinges and different boundary conditions, carrying a moving mass. The results are compared with a nonlinear dynamic finite element program, and found that they are in good agreement with each other. An approximate method, based on discrete element model for beams is presented and beams with internal hinges, variable boundary conditions and moving mass is investigated in this paper. Some problems were solved and the results are compared with a general-purpose nonlinear dynamic finite element program. Comparison of the results reveals a good agreement between the two methods. However, the proposed method is computationally very cheap and could be used on a personal computer.

P.K.Chatterjee et al [13] investigated the dynamic behavior of multi-span continuous beams under a moving load, by considering the effect of interaction between the vehicle and the bridge road, the torsion in the bridge due to eccentrically placed vehicles and the randomness of the surface irregularity of the road. The response of the bridge is obtained in the time domain by using an iterative procedure employed at each time step to take into account the non-linearity of the pavement-vehicle interactive force. The solution is made efficient by utilizing a continuous approach for determining the eigen functions of the bridge, and by obtaining the response at each iteration with the help of a few closed form expressions. The method is used to perform a parametric study to show the effect of some important parameters on the response of the bridge. A continuous approach has been used to find the dynamic response of a multi span continuous bride under a moving load, modeled as a single unsprung or sprung mass. The response has been obtained with respect to time and with due consideration of the non-linear effects of the bridge-vehicle interaction, of the torsion in the bridge and of simulated random irregularity of the bridge road. With the help of this method of analysis a parametric study has been conducted to investigate the influence of some important parameters on the dynamic behavior of the bridge.

Arturo O. Cifuentes [14] presented a combined finite element and finite difference technique to determine the response of a beam subjected to a moving mass. The technique used here is based on a Lagrange Multiplier formulation that allows one to represent the compatibility condition at the beam-mass interface using a set of auxiliary functions. This approach can be easily adapted to a standard finite element code. A simple technique based on the finite element method has been proposed to model the effect of a moving mass on a flexible structure. This technique employs a set of auxiliary functions to implement the compatibility condition at the beam-mass interface.

G.T. Michaltsos et al [15] deals with the linear dynamic response of a simply supported light (steel) bridge under a moving load-mass of constant magnitude and velocity including the effect of the centripetal and Coriolis forces, which always are neglected. The individual and coupling effect of these forces in connection with the magnitude of the velocity of the moving load are fully discussed using a solution method based on an author‟s older publication. A variety of numerical results allows us to draw important conclusions for structural design purposes.

G. T. Michaltsos [16] deals with the linear dynamic response of a simply supported elastic single span beam subjected to a moving load of constant magnitude and variable velocity. This analysis focuses attention on the effect of the acceleration or deceleration on the behavior of the beam under a single (one-axle) load, or a real vehicle model (two-axle load), while the influence of the damping of the beam is taken into account for this last model. A variety of numerical results allows us to draw important conclusions for structural design purposes.

G.T. Michaltsos [17] examined a lot of parameters that, usually, are not taken into account either during the design of a bridge or because some assumptions are supposed by the designer to hold true for the design and the calculations of a bridge. Some of the more serious of these parameters are the irregularities of the surface of the bridge's deck, the vehicle model selected, or the neglect of some forces that arise by the movement of the vehicle on the bridge. We will find out that the effect of those parameters is, sometimes, very remarkable. Ivica Kozar [18] solved numerically the P.D.E. for moving load has been with many benefits over closed solution (various boundary conditions, introduction of damping and discrete elements like springs and dashpots, additional supports and many more). Average acceleration method has been employed since direct use of finite differences had shown as being practically unusable. Numerical and analytical solutions have been compared. On the basis of the above numerical solution the procedures for finite element analysis have been developed. The result is a F.E. computer program for 2D dynamical analysis that is especially suitable for moving load analysis. Numerical model and measurements on a real bridge example have been successfully compared. As it can be seen through examples numerical approach to the problem of a moving load is quite suitable for engineering purposes: solutions are accurate and procedure based on average acceleration is robust. Further benefit of the numerical formulation is that various boundary conditions, damping, various ways of supports, changing forces can all be easily taken into analysis.

Derya Ozer et al[18] investigated dynamic behavior of Overhead Crane beams. An Bernoulli Euler thin beam is studied. Computerized analysis was carried out in SAP 2000. Dynamic response of the beam was obtained depending on the mass ratio of the load to the mass of the beam and the velocity of the load. Dynamic response of crane beams depends on velocity and mass of moving load. Since the position of the mass of moving load changes, it causes changes in the natural frequency of the system. While the load is moving, depending on the position of the mass of load the vibration of the system varies. For same mass ratio when the velocity of the load increases, the deflection of the beam goes higher. The dynamic behavior of the beam more affected from the velocity of load than mass ratio of the system. It is showed that carrying analysis in terms of only the midpoint deflection or midpoint stresses in engineering calculations of the beam systems is insufficient. It brings out more accurate results to take into account the mass and velocity of the moving load and dynamic properties of carrying system in dynamic analysis.

B. Mehri et al [19] presented the linear dynamic response of uniform beams with different boundary conditions excited by a moving load, based on the Euler-Bernoulli beam theory. Using a dynamic green function, effects of different boundary conditions, velocity of load and other parameters are assessed and some of the numerical results are compared with those given in the references. An exact and direct modeling technique is presented in this paper for modeling beam structures with various boundary conditions, subjected to a load moving at a constant speed. In order to validate the efficiency of the method presented, quantitative examples are given and results are compared with those available in the literature. In addition, the influence of a variation in the speed parameters of the system on the dynamic response of the beam was studied and the results were given in graphical and tabular form.

Jia-Jang Wu et al[20] in his paper presents a technique for using finite element packages for analyzing the dynamic response of structures to time-variant moving loads. To illustrate the method and for validation purposes, the technique is first applied to a simply supported beam subject to a single load moving along the beam. Finally, it is applied to the problem that initiated the work: calculation of the effects of two-dimensional motion of the trolley on the response of the base structure of a mobile gantry crane model. A technique for using standard finite element packages to analyze the dynamic response of structures to Time-variant moving loads has been developed. A computer Program has been written which calculates the time-variant external nodal forces on a whole structure, which provide the equivalent load to point forces that move around the structure. The calculation of the equivalent nodal forces to represent the moving loads has been performed by three approximate methods. In the first method, equivalent nodal forces and moments were calculated. This requires the shape functions for the element. The second method simply ignores the moment's calculated using the first method. The third method ignores any moment applied at the nodes at the outset and therefore did not require knowledge of the shape functions.

M. Dehestani et al[21] A traveling mass due to its mass inertia has significant effects on the dynamic response of the structures. According to recent developments in structural materials and constructional technologies, the structures are likely to be affected by sudden changes of masses and substructure elements, in which the inertia effect of a moving mass is not negligible. The transverse inertia effects have been a topic of interest in bridge dynamics, design of railway tracks, guide way systems and other engineering applications such as modern high-speed precision machinery process. In this study an analytical–numerical method is presented which can be used to determine the dynamic response of beams carrying a moving mass, with various boundary conditions. It has been shown that the Coriolis acceleration, associated with the moving mass as it traverses along the vibrating beam shall be considered as well. Influences regarding the speed of the moving mass on the dynamic response of beams with various boundary conditions were also investigated. Results illustrated that the speed of a moving mass has direct influence on the entire structural dynamic response, depending on its boundary conditions. Critical influential speeds in the moving mass problems were introduced and obtained in numerical examples for various boundary conditions.

S.A.Q. Siddiqui et al[22] developed a technique to obtain numerical solution over a long range of time for non-linear multi-body dynamic systems undergoing large amplitude motion. The system considered is an idealization of an important class of problems characterized by non-linear interaction between continuously distributed mass and stiffness and lumped mass and stiffness. This characteristic results in some distinctive features in the system response and also imparts significant challenges in obtaining a solution. In this paper, equations of motion are developed for large amplitude motion of a beam carrying a moving spring–mass. The equations of motion are solved using a new approach that uses average acceleration method to reduce non-linear ordinary differential equations to non-linear algebraic equations. The resulting non-linear algebraic equations are solved using an iterative method developed in this paper. Dynamics of the system is investigated using a time-frequency analysis technique.

CHAPTER 3

-RESPONSE OF BEAM TO A DYNAMIC LOAD-

3. RESPONSE OF BEAM TO A DYNAMIC LOAD

The problem of moving loads on structures was first considered in the early Nineteenth century when the traversing of bridges by locomotives was analyzed, this has been followed by a considerable amount of research on this topic. The purpose of dynamic analysis is to know the structural behavior under the influence of various loads and to get the necessary information for design such as deformation, moments and dynamic forces etc. Structural analysis is classified in to static and dynamic analysis. Static analysis deals with load which is time independent. But in dynamic analysis magnitude, direction and position of mass change with respect to time. Important dynamic loads for vibration analysis of bridge structure are vehicle motion and wave actions i.e. earthquake, stream flow and winds. The effect of a moving body travelling over a bridge can be solved with various assumptions such as

- The bridge has insignificant mass compared to the moving body,
- The moving body's mass is slight compared to the bridge,
- The whole system can be dynamically analyzed in full when considering the effects of the masses of both the bridge and body.

3.1 Simply Supported Beam

A simply supported beam has a hinged connection at one end and roller connection at other end. Calculating the natural frequency of a system is necessary to find how the system will act when just disturbed and left. Vibration analysis of a Simply Supported beam system is essential as it can help us examine a number of real life systems. The following few examples are taken for a simply supported beam that helps us to design them. Now in the building of sky scrapers, transmission towers etc. the truss elements could be simply supported depending on their construction. Also in the design process of bridges simply supported beam analysis plays a significant role. In the automotive industry, the leaf spring suspension system can be assumed to be a simply supported beam. In machines and machine tools, there are a number of systems which can be studied taking the supports to be simply supported.

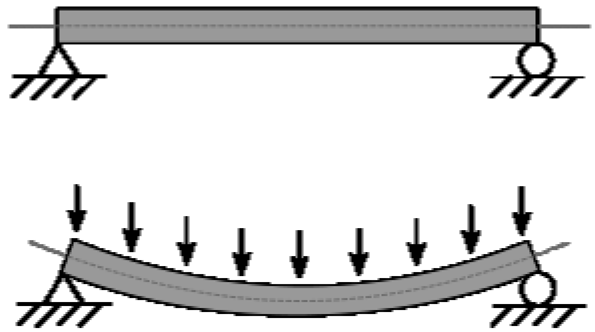


Fig.3.1.Simply Supported Beam

The numerical theory on simply supported beam has many applications for the calculations related to long span railway and high way bridges. In most cases, the mass of the simply supported beam is higher than the mass of the vehicles. So that, if vibration of moving body on own springs is neglected, the effects of moving mass may more or less be replaced by the effects of moving forces.

In a simply supported beam structure, moments are not transfer through the support so their structural type is known as simply supported.

3.2 Cantilever Beam

A beam with a laterally and rotationally fixed support at one end and with no support at the other end is called a cantilever beam. Cantilever construction allows for overhanging structures without external bracing. Cantilevers can also be constructed with trusses or slabs. Cantilever bridges are built with cantilevers, structures that project horizontally into the space and supported on only one end. For small bridges the cantilevers may be simple beams, though large cantilever bridges designed to handle the road or rail traffic use trusses.

A simple cantilever bridge can be designed by two cantilever arms projecting from opposite sides of the obstacle to cross and meet at the center. In a common variant, the suspended span and the cantilever arms do not meet in the center instead of that they support a central truss bridge which rests on the ends of the cantilever arms. The suspended span can be built off-site and lifted into place or constructed in place using special traveling supports. In the recent years, tests are conducted on different types of beam bridges and the result obtained with more correctness with large displacements in the case of cantilever bridges than other types. For the behavior in dynamic condition of the cantilever beam, the analysis on frequency has to be made.

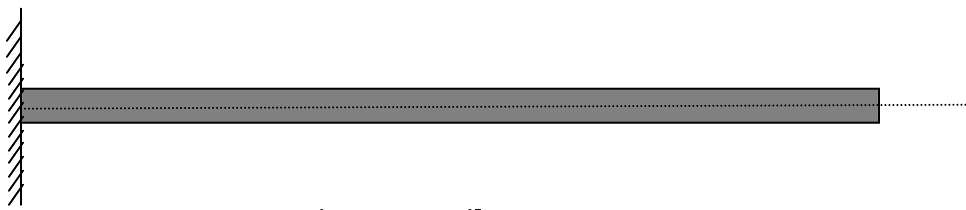


Fig.3.2.Cantilever Beam

CHAPTER 4

-DYNAMIC RESPONSE OF BEAM SUBJECTED TO MOVING MASS-

4. DYNAMIC RESPONSE OF BEAM SUBJECTED TO MOVING MASS

4.1. Introduction:

In the present research work, dynamic response of beams such as simply supported and cantilever beams subjected to moving mass under various conditions has been calculated by using theoretical analysis. To achieve the beam response it is a necessary to make problem formulation for the beams under moving mass. The response of both types of beams is shown in the subsequent sections.

4.2. Theoretical Analysis:

A. Problem Formulation:

The differential equation of a beam, which is assumed as a Euler-Bernoulli beam subjected to a point force is given by

$$EI\ \frac{\partial^4 w(x,t)}{\partial x^4} + m\frac{\partial^2 w(x,t)}{\partial t^2} = F\delta(x-u) \tag{1}$$

Where E is Young's modulus, I is moment of area of the beam cross-section, E is Young's modulus, m is the mass per unit length of the beam, x is the axial co-ordinate, t is the time, $w(x,t)$ is the transverse displacement of the beam, F is the applied point force and $\delta(x-u)$ is the Dirac delta function.

The Dirac delta can be loosely thought of as a function on the real line which is zero everywhere except at the origin, where it is infinite,

$$\delta(x-u) = \begin{cases} +\infty & x = 0 \\ 1 & x \neq 0 \end{cases} \tag{2}$$

And which is also constrained to satisfy the identity

$$\int_{-\infty}^{\infty} \delta(x)\, dx = 1 \tag{3}$$

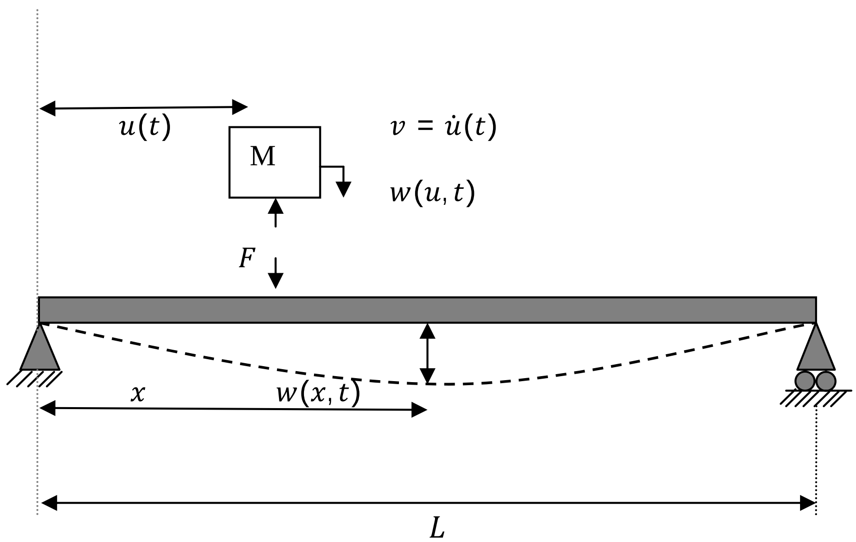


Fig.4.1. A mass traversing on a simply supported beam with constant velocity

The boundary conditions for the simply supported beam are

$$w(x,t) = \frac{\partial^2 w(x,t)}{\partial t^2} = 0 \qquad \text{At } x = 0 \textit{ and } L, \tag{4}$$

And the initial conditions for the simply supported beam are

$$w(x,0) = \frac{\delta w(x,0)}{\delta t} = 0 \tag{5}$$

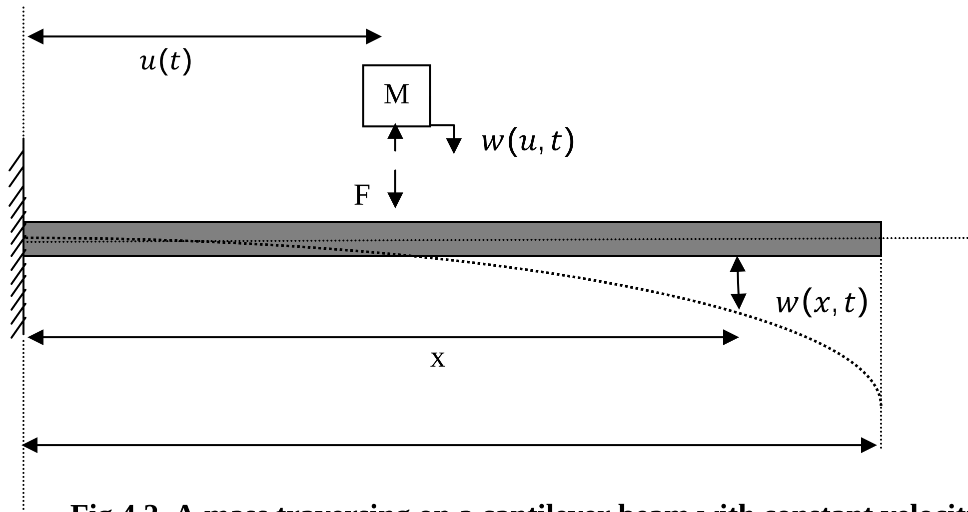


Fig.4.2. A mass traversing on a cantilever beam with constant velocity

The boundary conditions for cantilever beam are

1. $w(0) = 0$
2. $\dot{w}(0) = 0$
3. $\ddot{w}(L) = 0$
4. $\dddot{w}(L) = 0$ (6)

Referring to above Figures, F is the reaction force exerted by the mass M on the beam. Newton's second law when applied to the mass M we find,

$$F = M(\, g - \frac{\partial^2 \beta}{\partial t^2}\,), \tag{7}$$

Here β is the transverse displacement of the mass and g is the acceleration due to gravity. Therefore, in the above and here in after we use the notation

$$\beta(t) = w(x,t)\big|_{x=u.} \tag{8}$$

Here, the solution of the differential equation is going to be obtained by using Dynamic green's function. Hence, if (x,u) is the dynamic Green function, then the solution of equation (1) is of the form

$$w(x,t) = G(x,u)F \tag{9}$$

Where $G(x,u)$ is the solution of the differential equation

$$\frac{\partial^4 w(x)}{\partial x^4} - q^4 w(x) = \delta(x-u), \tag{10}$$

Where q is the frequency parameter and it is given by

$$q^4 = \frac{\boldsymbol{\omega^2 m}}{\boldsymbol{EI}} \tag{11}$$

In which ω is the circular frequency that gives the motion of the mass and is equal to $\pi V/L$.

The solution of equation (8) is assumed in the form

$$G(x,u) = \begin{cases} A_1 cos(qx) + A_2 \sin(qx) + A_3 \cosh(qx) + A_4 \sinh(qx) & 0 \le x \le u \\ B_1 cos(qx) + B_2 \sin(qx) + B_3 \cosh(qx) + B_4 \sinh(qx) & u \le x \le L \end{cases} \tag{12}$$

The eight constants $A_1, \ldots, A_4$ and $B_1, \ldots, B_4$ are evaluated such that the Green function $G(x,u)$ satisfies the following conditions for a simply supported beam,

(a) Two boundary conditions at each end of the beam depending on the type of end support-for a simply supported beam,

$$G(0,u) = G(L,u) = G''(0,u) = G''(L,u) \tag{13}$$

Where the prime indicates a derivative with respect to x;

(b) Continuity conditions of displacement, slope and moment at $x = u$, i.e.

$$G(u^+,u) = G(u^-,u), \quad G'(u^+,u) = G'(u^-,u), \quad G''(u^+,u) = G''(u^-,u), \tag{14}$$

(c) Shear force discontinuity of magnitude one at $x = u$, i.e.,

$$EI[G'''(u^+,u) - G''(u^-,u)] = 1 \tag{15}$$

The Green function determined for a simply supported beam by the above procedure is given by

$$G(x,u) = \frac{1}{2EIq^3\sin(qL)\sinh(qL)}\begin{cases} g(x,u) & 0 \le x \le u \\ g(u,x) & u \le x \le L \end{cases} \tag{16}$$

Where

$$g(x,u) = \sinh(qL)\sin(qx)\sin(qL-qu) - \sin(qL)\sinh(qx)\sinh(qL-qu) \tag{17}$$

And $g(x,u)$ is obtained by switching x and u in $g(x,u)$. This follows from the fact that $G(x,u)$ must be symmetric to satisfy Maxwell-Rayleigh reciprocity law.

It is also noticed that when q is equal to zero, the expression given by equation (14) reduces to the static Green function for a simply supported beam,

$$\begin{aligned} \lim_{q\to 0} G(x,u) &= \frac{L^3}{6EI}\left(1-\frac{u}{L}\right)\left(\frac{x}{L}\right)\left[2\frac{u}{L}-\left(\frac{x}{L}\right)^2-\left(\frac{u}{L}\right)^2\right] \quad 0 \le x \le u \\ \lim_{q\to 0} G(x,u) &= \frac{L^3}{6EI}\left(1-\frac{x}{L}\right)\left(\frac{u}{L}\right)\left[2\frac{x}{L}-\left(\frac{u}{L}\right)^2-\left(\frac{x}{L}\right)^2\right] \quad u \le x \le L \end{aligned} \tag{18}$$

Similarly the Green function determined for cantilever beam is given by

$$G(x,u) = \frac{1}{2EIq^3\Delta}\begin{cases} g(x,u) & 0 \le x \le u \\ g(u,x) & u \le x \le L \end{cases} \tag{19}$$

Where

$$g(x,u) = D_1(\cos qx - \cosh qx) + D_2(\sin qx - \sinh qx) \tag{20}$$

Here

$$\Delta = 2(1 + \cos qL \cosh qL)$$

$$D_1 = (\cos qL + \cosh qL)(\sin z + \sinh z) - (\sin qL + \sinh qL)(\cos z + \cosh z)$$

$$D_2 = (\sin qL - \sinh qL)(\sin z + \sinh z) + (\cos qL + \cosh qL)(\cos z + \cosh z)$$

And $g(x,u)$ is obtained by switching x and u in $g(x,u)$. This follows from the fact that $G(x,u)$ must be symmetric to satisfy Maxwell-Rayleigh reciprocity law.

It is suitable to change the variable by using relationship $u = u(t)$, so that

$$\frac{d\beta}{dt} = \frac{d\beta(t)}{du}\frac{du}{dt} = v\ \frac{d\beta(u)}{du},$$

$$\frac{d^2\beta}{dt^2} = v^2\frac{d^2\beta}{du^2} \tag{21}$$

Where $$\beta(u) = w(x,u)\big|_{x=u}. \tag{22}$$

Eliminating F between equation (8) and (10) and making use of equation (16) yields

$$w(x,u) = G(x,u)\,M\left[g - v^2\ \frac{d^2\beta}{du^2}\right], \tag{23}$$

Equation (23) is a second order differential equation, which specifies the beam deflection at position x caused by the load at position u.

B. Computational Algorithm:

As the closed form solution of equation (23) is not known, we should search an approximate solution in which the derivatives are replaced by their finite difference approximation. If the beam is divided into $(N-1)$ intervals each of length h. The discretized equation (23) is

$$w(x_i,u) = G(x_i,u)\,M\left[g - v^2\ \frac{d^2\beta}{du^2}\right], \tag{24}$$

Here the subscript i refers to any one of the N discrete station points. It is to be noted that equation (23) is explicit in u and implicit in time$t$. Without loss of generality, the variable u that represents the location of mass is discretized in the same way. Therefore the increment of u can also taken to be of length h. This incrementation is equivalent to using time intervals that are the length of time required to travel from any one of the N stations to the next adjacent one.

Now we are left with the task to use finite divided difference formula to represent the u derivative in equation (18). The Houbolt method is chosen, since u is equivalent to the time variable. Therefore letting $f(u)$ be any sufficiently smooth function, the approximate formula is

$$\frac{d^2 f}{du^2} = \frac{1}{h^2}\sum_{k=0}^{3} a_k\, f\left(u_{j-k}\right) + O(h^2) \tag{25}$$

Where u_j indicates that the mass is at the jth station point. The coefficient a_k are $a_0 = 2$,

$a_1 - 5$, $a_2 = 4$, $a_3 = -1$.

Application of equation (19) to equation (18) results in the following set of algebraic equations:

$$h^2 w\left(x_i, u_j\right) = G\left(x_i, u_j\right) M\left[h^2 g - v^2 \sum_{k=0}^{3} a_k\, w\left(x_{j-k}, u_j\right)\right], \qquad i, j = 1,2, \dots\dots, N \tag{26}$$

Where use has been made of equation (16) with remark that

$$\beta\left(u_j\right) = w\left(x, u_j\right)\Big|_{x=u_j} = w(x_j, u_j), \tag{27}$$

Equation (20) is same as equation (11), neglecting the numerical integration in that equation, replacing the influence coefficient by the dynamic Green function, and setting $\dot{v} = 0$.

Equation (20) can be expressed in matrix form:

$$[h^2[I] + a_0 v^2 [G] M [P_{ij}]]\{w_{u_j}\} = M[G]\ [h^2 g\{\Delta_{u_j}\} - v^2 \sum_{k=1}^{3} a_k \left[P_{j,j-k}\right]\{w_{u_{j-k}}\}], \tag{28}$$

Where $[I]$ is the identity matrix and

$$\{w_{u_j}\} = \{\, w(x_1, u_j), w(x_2, u_j), \dots \dots \dots w(x_N, u_j)\}^T \tag{29}$$

$$\{\Delta_{u_j}\} = \{000 \dots .\underset{\substack{\uparrow \\ jth\ column}}{010} \dots 000\}^T \tag{30}$$

$$[G] = \begin{bmatrix} G(x_1, u_1) & \dots . & G(x_1, u_N) \\ \dots . & & \dots . \\ \dots . & \dots . & \dots . \\ \dots . & & \dots . \\ G(x_N, u_1) & \dots . & G(x_N, u_N) \end{bmatrix},$$

$$[P_{i,j-k}] = \begin{array}{c} (j-k)th\ \ column \\ \downarrow \\ \begin{bmatrix} 0 & \dots \dots \dots \dots & 0 \\ 0 & \dots \dots \dots \dots & 0 \\ 0 & ..0\ 1\ 0 \dots & 0 \\ 0 & \dots \dots \dots \dots & 0 \\ 0 & \dots \dots \dots \dots & 0 \end{bmatrix} \end{array} \begin{array}{l} \\ \\ \\ \\ \leftarrow jth\ column \\ \\ \\ \end{array} \tag{31}$$

Equation (22) describes the transverse displacement of the beam when the mass is at any station j. In order to include the initial conditions (3), which describes the displacement and slope of beam while in motion; one looks up to the change of variables introduced earlier. Accordingly, the initial conditions correspond to the mass being at the initial station point $j = 0$. For this station, equation (21) requires a value of $\{w_{u_j}\}$, where $k \leq 0$. These values may be set equal to zero.

Equation (22) may be placed into non-dimensional form so that the numerical results presented are applicable for large combinations of system parameters. This is achieved by letting w_{st} be the scaling factor for the transverse displacement, where w_{st} is the static deflection at the beam mid-span due to the weight of mass, and by letting T be the time scale, where T is the period of lowest vibration mode of the beam. Thus

$$w_{st} = \frac{MgL^3}{48EI} \quad , \qquad T = \frac{2L^2}{\pi}\sqrt{\frac{m}{EI}} \tag{32}$$

The appropriate non-dimensional quantities can be

$$\hat{w} = \frac{w}{w_{st}} \ , \qquad \hat{G} = \frac{EI}{L^3}G. \tag{33}$$

Using equation (26) the non-dimensional form of equation (21) is given by

$$\left[[I] + \gamma a_0[\hat{G}][P_{ij}]\right]\left\{\hat{w}_{u_j}\right\} = [\hat{G}]\left[48\left\{\Delta_{u_j}\right\} - \gamma\sum_{k=1}^{3} a_k\left[P_{j,j-k}\right]\left\{w_{u_{j-k}}\right\}\right], \tag{34}$$

Where the non-dimensional parameter γ depends on the mass ratio M/mL, the speed ratio α(defined below), and the number of segments, as given by

$$\gamma = \pi^2\left(\frac{M}{mL}\right)\left(\frac{L}{h}\right)^2\left(\frac{v}{v_{cr}}\right)^2 \tag{35}$$

The speed ratio α is defined as

$$\alpha = \frac{vL}{\pi}\sqrt{\frac{M}{EI}} \tag{36}$$

And the critical speed v_{cr} is defined as

$$v_{cr} = \frac{2L}{T} = \frac{\pi}{L}\sqrt{\frac{EI}{m}} \tag{37}$$

The case $\frac{v}{v_{cr}} = 1$ corresponds to resonance with the fundamental mode when the load is a constant force.

4.3. Numerical Analysis:

For the purpose of numerical analysis of beams, the parameters are selected corresponding to the data used in reference [15]. The displacement of the beam at any time can be obtained from equation (22).The values of the dynamic response have been calculated for two types of materials for both simply supported and cantilever beams. The materials considered are Aluminum and steel, which are mostly used in the construction of the bridges. For these beams the displacements at the end or middle points and at any arbitrary point on the beam are found for different values of mass and its speed. In equation .16, In order to evaluate approximate response of beams the derivatives are replaced by finite difference approximation.

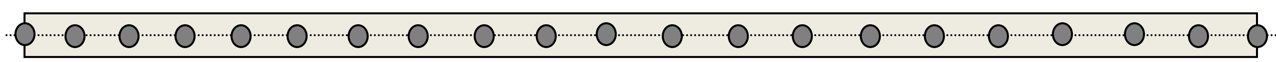


Fig 4.3 Beam with 21 stations

Here the beam is divided into 20 intervals of length 2.5 m each for the total span of 50 m. So the number of discrete station points located on the beam are 21 stations, therefore displacement of the beams can be found at 21 points while mass is moving on the span.

For Numerical analysis, the beam specimen and specific locations for displacement on the beam are selected as follows.

I) Beam specimens: Length = 50 m.,
Moment of inertia=1.042 m4,
Mass per unit length of beam=4800 kg/m

II) Moving Mass: M = 25000,50000 kg

III) Velocity of the Moving Mass: v = 8.45, 42.29, and 84.59 m/s

The deflections of the beams are calculated and the results are presented in the following graphs.

Case.1

1. Beam type: Simply Supported Beam
2. Material: Aluminum (E=69 GPa)
3. Weight of the moving mass: 25000 kg

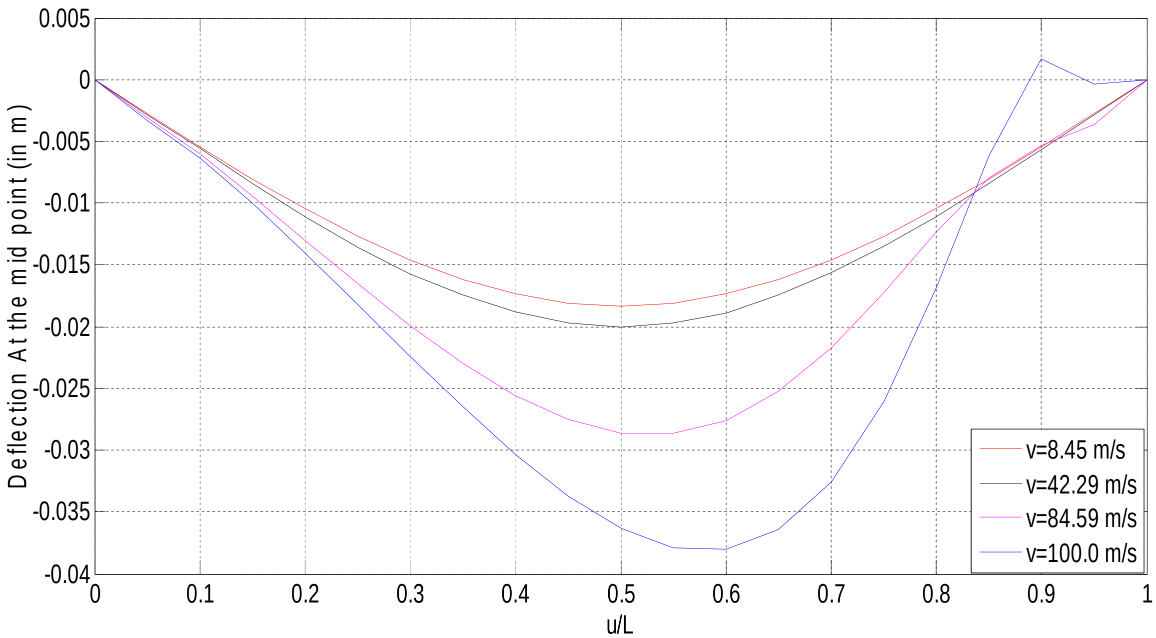


Fig.4.4.Deflections of Simply Supported Beam at the midpoint for different Velocities as shown

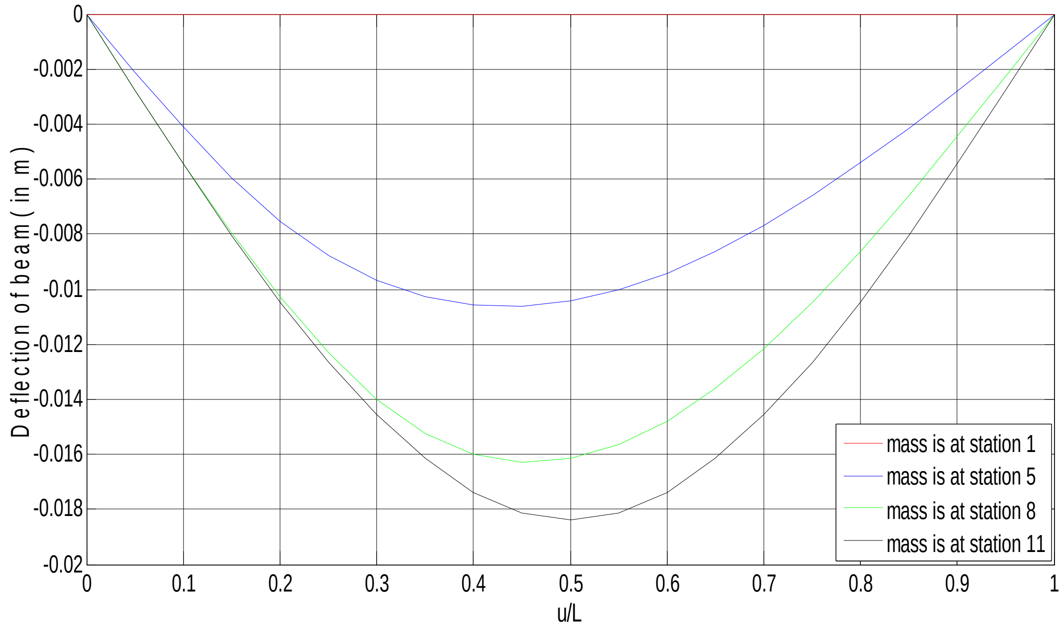


Fig.4.5.Shape of the beam when the mass is moving through different stations at velocity of 8.45m/s

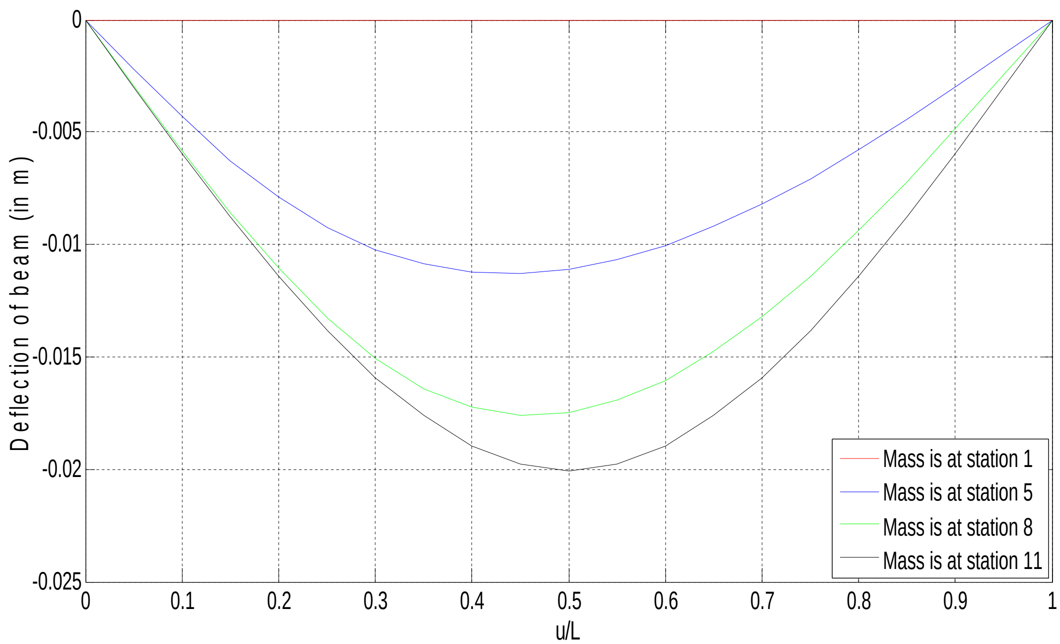


Fig.4.6.Shape of the beam when the mass is moving through different stations at velocity of 42.29 m/s

Case.2

1. Beam type: Simply Supported Beam
2. Material: Steel (E=200 GPa)
3. Weight of the moving mass: 25000 kg

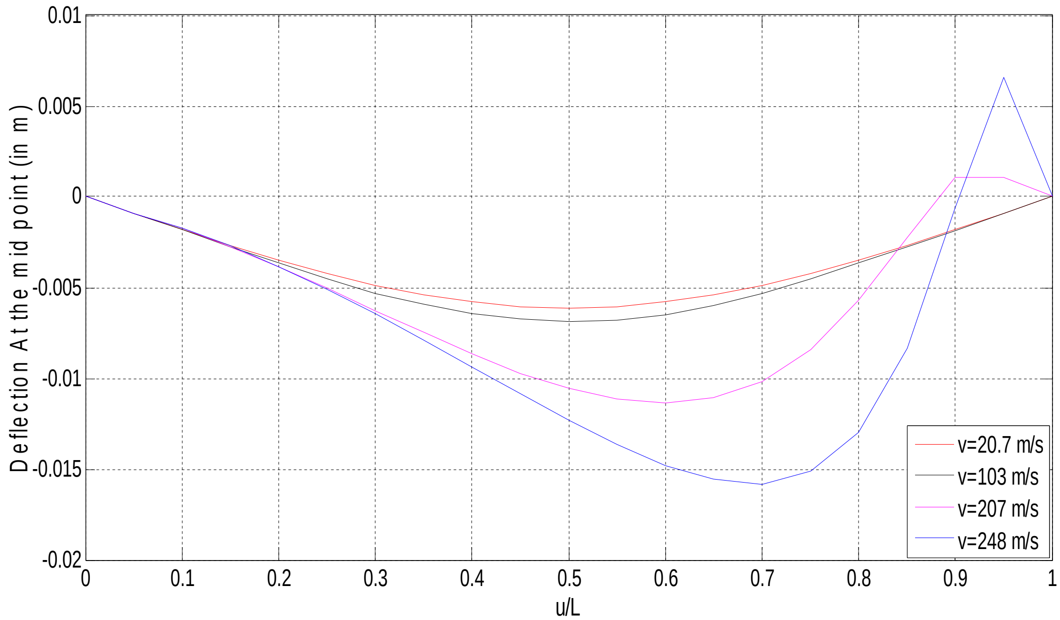


Fig.4.7.Deflections of Simply Supported Beam at the midpoint for different Velocities as shown

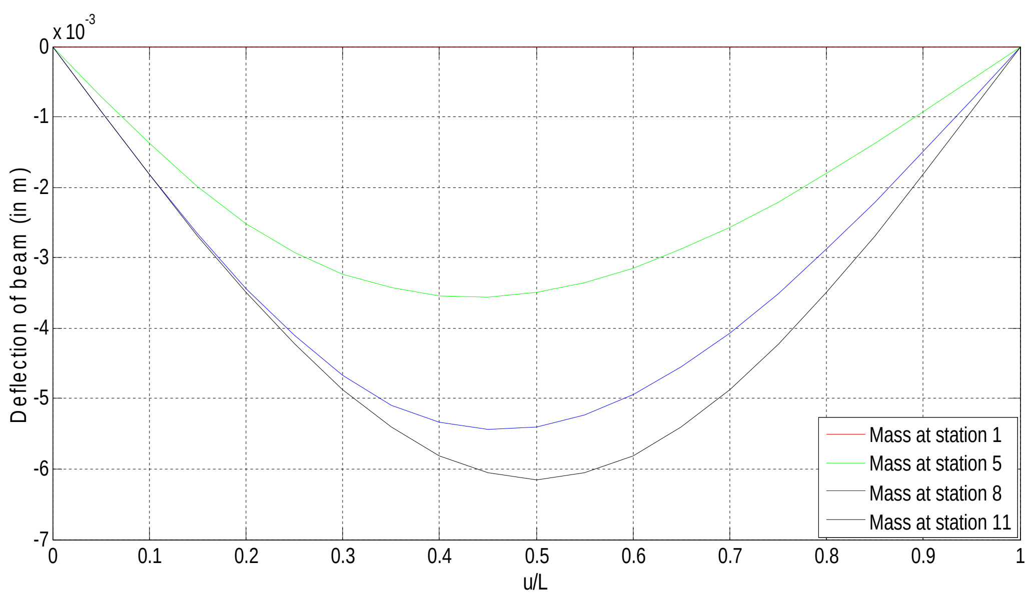


Fig.4.8.Shape of the beam when the mass is moving through different stations at velocity of 20.7 m/s

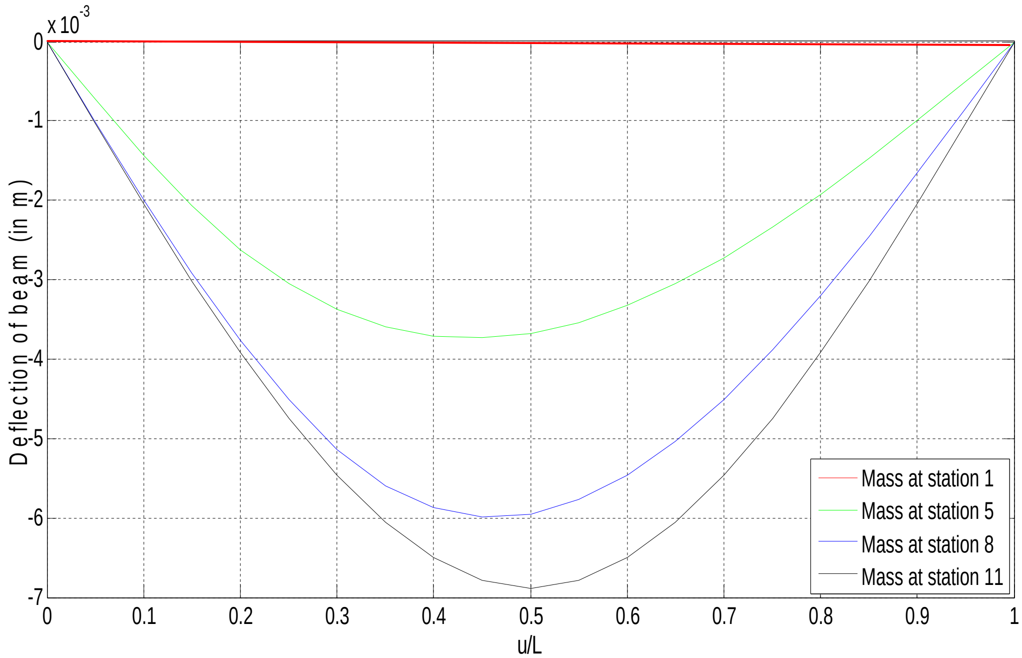


Fig.4.9.Shape of the beam when the mass is moving through different stations at velocity of 103.5 m/s

Case.5

1. Beam type: Cantilever Beam
2. Material: Steel (E=200 GPa)
3. Weight of the moving mass: 25000 kg

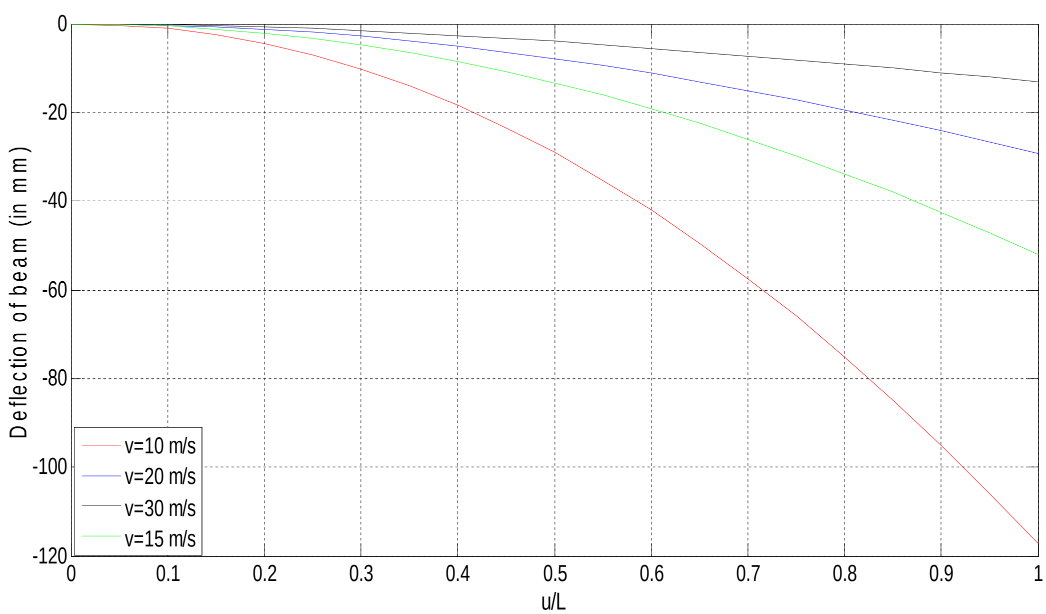


Fig.4.10.Deflections of Cantilever Beam at the end point for different Velocities as shown

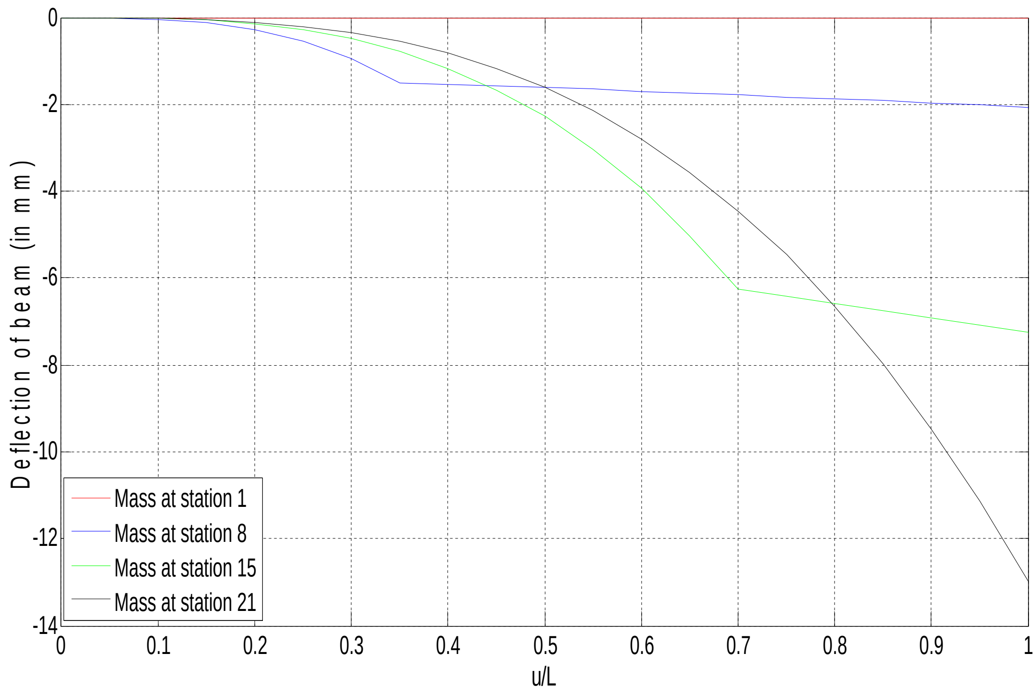


Fig.4.11.Shape of the beam when the mass is moving through different stations at velocity of 30 m/s

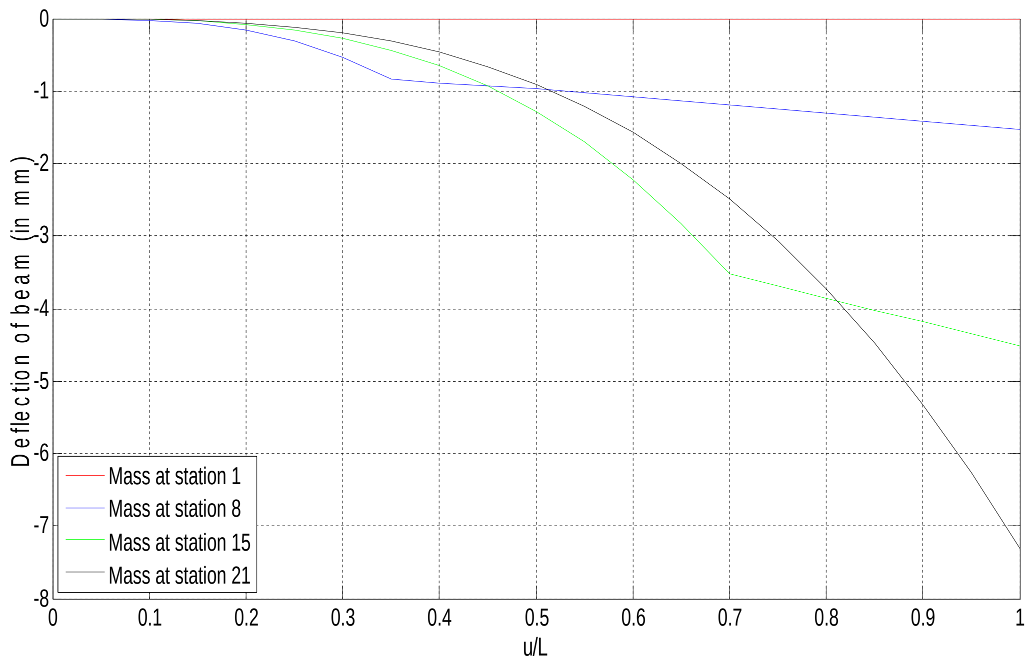


Fig.4.12.Shape of the beam when the mass is moving through different stations at velocity of 40 m/s

Case.6

1. Beam type: Cantilever Beam
2. Material: Steel (E=200 GPa)
3. Weight of the moving mass: 50000 kg

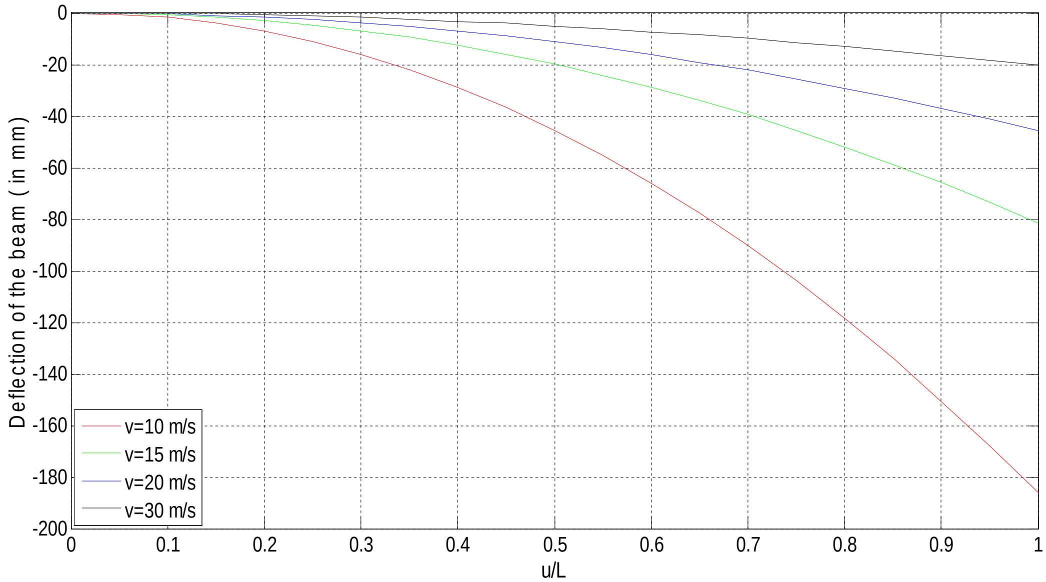


Fig.4.13.Deflections of Cantilever Beam at the end point for different Velocities as shown

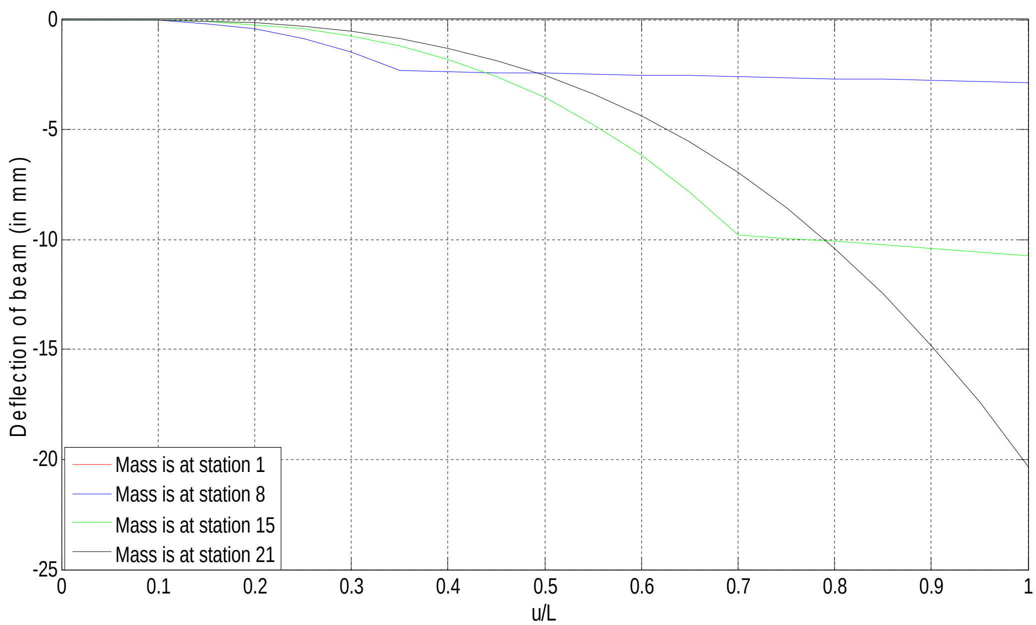


Fig.4.14.Shape of the beam when the mass is moving through different stations at velocity of 30 m/s

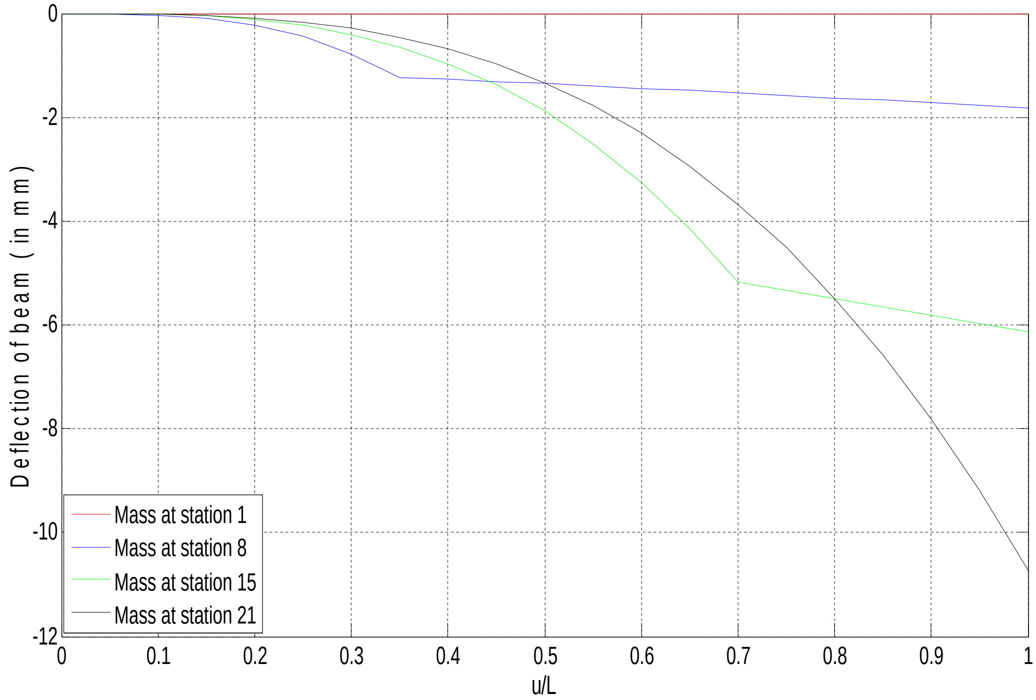


Fig.4.15.Shape of the beam when the mass is moving through different stations at velocity of 40 m/s

CHAPTER 5

-EXPERIMENTAL WORK-

5.1 Experimental Setup and Procedure:

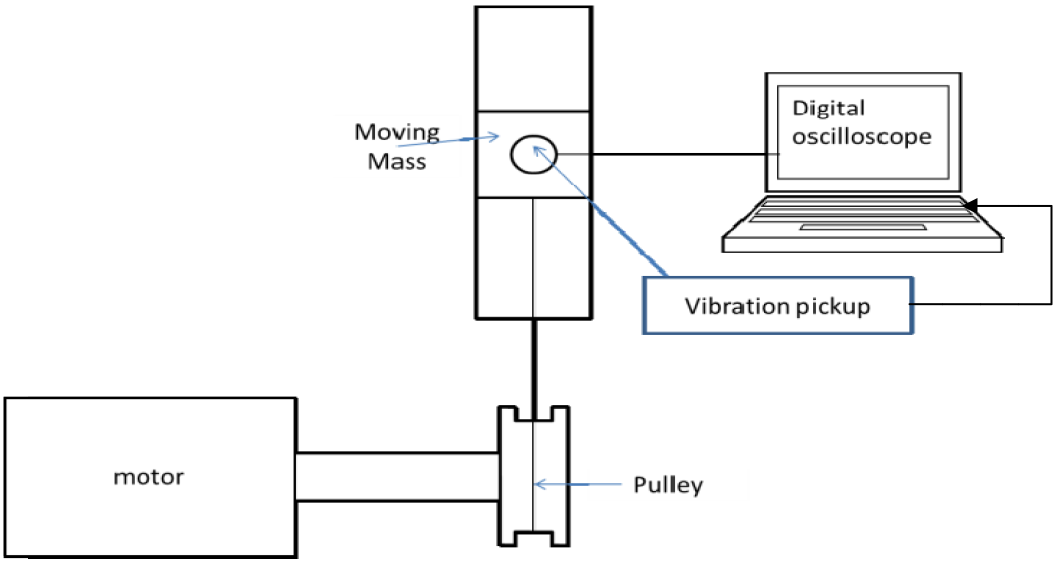


Fig 5.1 Experimental setup showing different equipments.

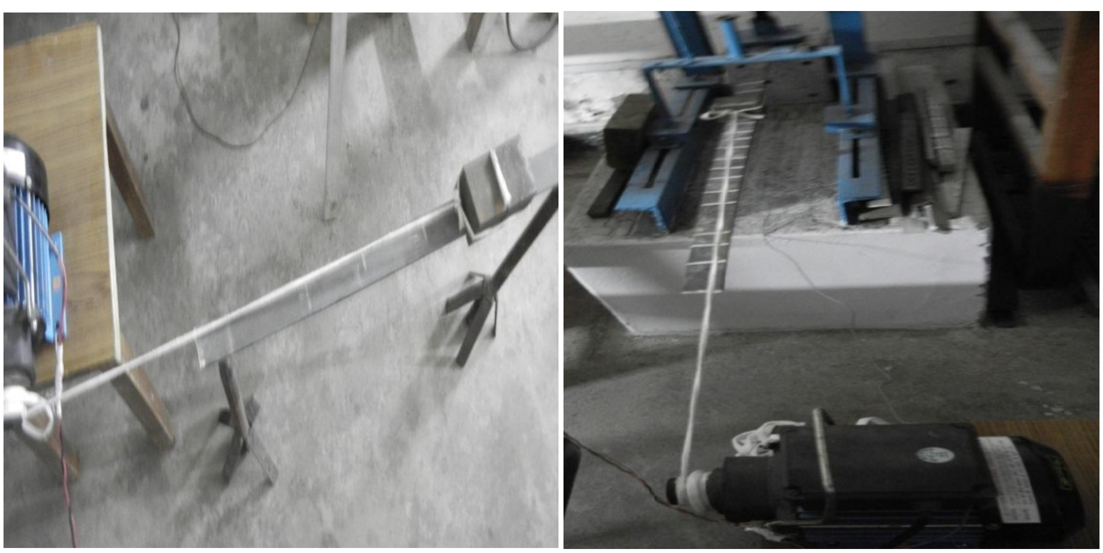


Fig.5.2 Experimental setup in dynamics lab at NIT Rourkela for Simply support and Cantilever beam.

Experimental setup for a simply supported and cantilever beam is made which are shown in above figure. The beam of 1m is divided in to 20 stations and a vibration pickup is attached at lower the side of beam (attached at mid-span in case of simply supported and at the end of beam in cantilever case).vibration pickup is connected to the digital oscilloscope which shows the wave pattern generated on the screen. Amplitude of vibration or deflection of beam can be recorded from the oscilloscope. In simply supported beam amplitude of deflection at mid span is recorded while mass moving through different stations. But in case of cantilever beam deflection at end point of beam is recorded while mass traversing through different stations. Dynamic response of beam is studied for different speed and mass.

5.2 Equipments Used

- vibration pick up or transducer
- digital oscilloscope(Tektronix 4000 series)
- U-shape iron block(used as moving mass)
- Aluminum and structural steel beams
- 0.5 HP motor with a pulley attached on shaft.

5.3 Equipment's Description

❖ Vibration pick up:

It is an electro-mechanical transducer which can able to convert the mechanical vibration generated into electrical signals which can be displayed on the screen of digital oscilloscope. Depending upon the nature of work vibration pickups are of different types i.e. accelerometer, displacement pick up, velocity picks up.

Fig 5.3 Vibration pick up used during experiment.

❖ Digital phosphor oscilloscope:

It’s an electronic device used to display the shape of electrical signals in the form of a graph in terms of voltage-time co-ordinate from which the amplitude of deflection of beam can be recorded. Some key features of the Tektronix 4000 series digital oscilloscope are:

- 1GHz,500MHz,and 350MHz bandwidths
- 2 and 4 channel models.
- 35000 waveforms/second display rate.
- USB and Compact Flash available for quick and easy storage.

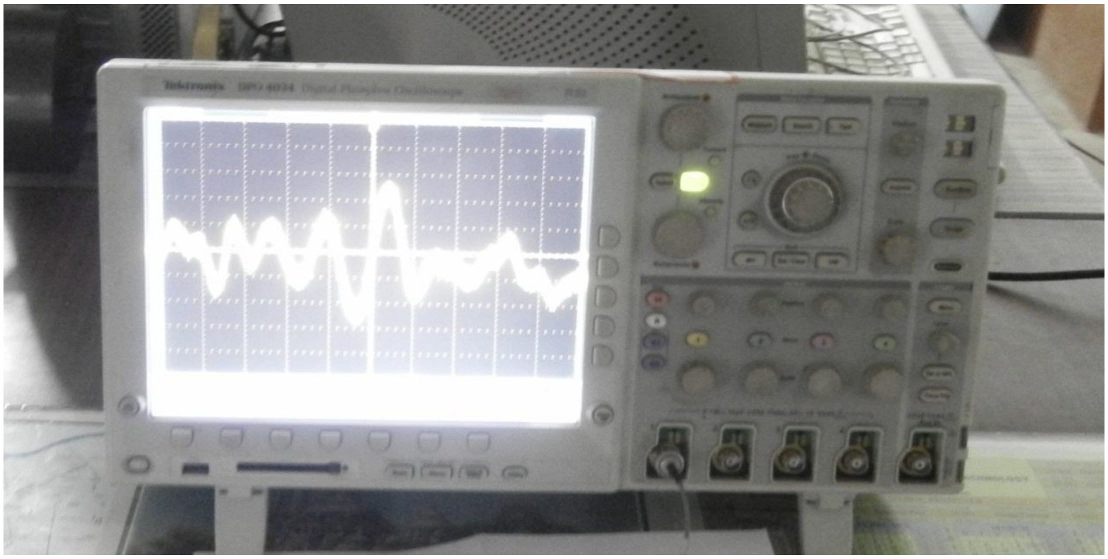


Fig.5.4 Tektronix 4000 series digital oscilloscope.

❖ U-shape iron block:

Two u-shapes iron block of 1.8kg and 0.9kg are made with the help of CNC machine in workshop of NIT Rourkela.

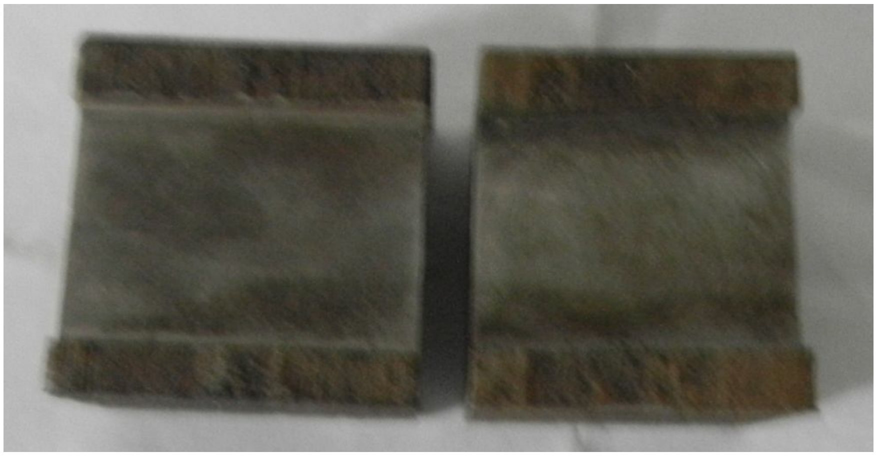
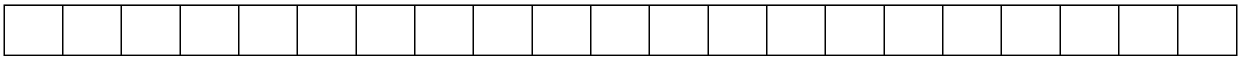


Fig.5.5 Masses of 0.9kg and 1.8 kg used as moving masses.

5.4. EXPERIMENTAL RESULTS

5.4. a. FOR SIMPLY SUPPORTED BEAM

Beam of 1m is divided in to 21 stations with each interval of 0.05m.



Case I: structural steel beam

1. Beam dimension and specification:

   Length (L) =1m, breadth (b) =5cm, width (d) =0.5cm

   E=200GPa

   $I = \frac{bd^3}{12} = 5.2 \times 10^{-10} m^4$

   Mass per unit length (m) =3kg/m
2. Moving mass(M): 0.9kg,1.8kg
3. Velocity of moving mass(v): 1, 2.5, 5,and 7 m/s

   Or corresponding velocity ratio (α) =0, 0.05, 0.125, 0.25, 0.375

   Here, $\alpha = {}^{v}/_{v_{cr}} \quad and \quad v_{cr} = \frac{\pi}{L}\sqrt{\frac{EI}{m}}$

Table for deflection of beam at mid span for moving mass traversing through different station and corresponding graphs are given below for different mass and speed.

x	y(α=0)	y(α=0.05)	y(α=0.125)	y(α=0.25)	y(α=0.375)
0	0	0	0	0	0
0.05	-0.1095	-0.1592	-0.1278	-0.1431	-0.1367
0.1	-0.296	-0.1953	-0.2919	-0.2809	-0.266
0.15	-0.3155	-0.4371	-0.439	-0.4355	-0.4198
0.2	-0.568	-0.5694	-0.4771	-0.5923	-0.588
0.25	-0.5875	-0.6898	-0.7021	-0.7413	-0.7625
0.3	-0.792	-0.5951	-0.9117	-0.8752	-0.9355
0.35	-0.8785	-0.8824	-0.8032	-0.9887	-1.0996
0.4	-0.944	-0.9486	-0.9732	-1.0767	-1.2464
0.45	-0.9855	-0.9905	-1.0178	-1.1344	-1.3661
0.5	-1.04	-1.0051	-1.0335	-1.117	-1.4473
0.55	-0.9855	-0.9905	-1.0179	-1.1407	-1.4773
0.6	-0.844	-0.9486	-0.9735	-1.0864	-1.447
0.65	-0.8785	-0.8824	-0.9036	-0.8974	-1.3482
0.7	-0.792	-0.7951	-0.8118	-0.8791	-1.1739
0.75	-0.6875	-0.5898	-0.7016	-0.7394	-0.7229
0.8	-0.568	-0.5694	-0.4666	-0.5892	-0.6108
0.85	-0.4365	-0.4372	-0.4403	-0.4397	-0.2927
0.9	-0.296	-0.2761	-0.2966	-0.2952	-0.0929
0.95	-0.1495	-0.1494	-0.1086	-0.1479	-0.1129
1	0	0	0	0	0

Table 1.Deflection of structural steel beam at mid span for mass 0.9kg at different values of α.

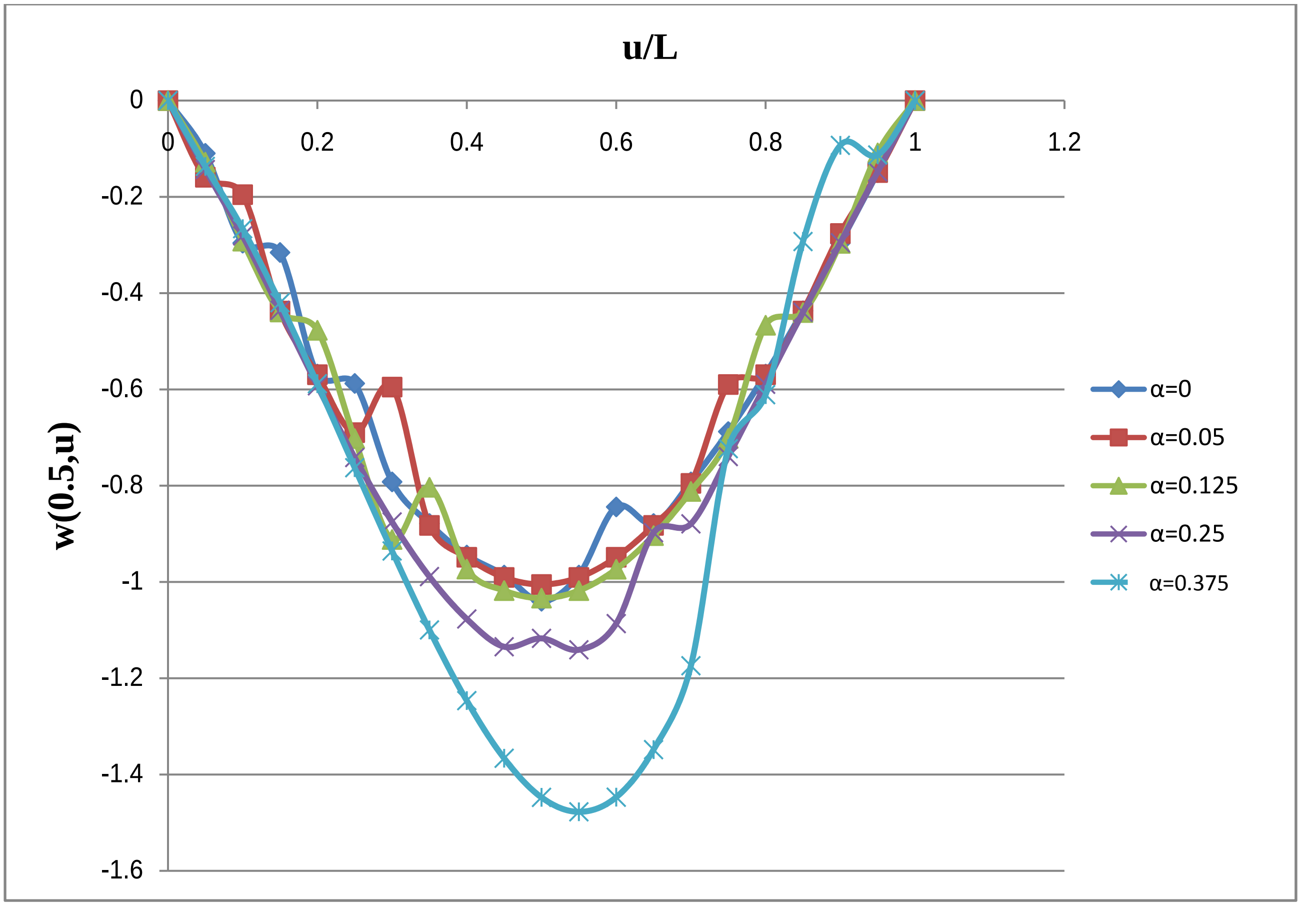


Fig.5.6 Mid-span deflection of beam traversed by a moving mass when $M/_{mL} = 0.3$

x	y(α=0)	y(α=0.25)	y(α=0.375)	y(α=0.05)	y(α=0.125)
0	0	0	0	0	0
0.05	-0.1295	-0.1196	-0.1122	-0.1486	-0.1439
0.1	-0.296	-0.2532	-0.2172	-0.2939	-0.2836
0.15	-0.2365	-0.399	-0.3463	-0.4365	-0.4322
0.2	-0.568	-0.5564	-0.4947	-0.5695	-0.5753
0.25	-0.7875	-0.7166	-0.6582	-0.6903	-0.6057
0.3	-0.792	-0.6718	-0.6329	-0.7962	-0.8197
0.35	-0.6785	-1.0142	-1.0145	-0.8841	-0.3147
0.4	-0.944	-1.1361	-1.1977	-0.9508	-0.99
0.45	-0.9855	-1.3286	-1.3753	-0.993	-1.035
0.5	-1	-1.2822	-1.5371	-1.0078	-1.0525
0.55	-0.9855	-1.2874	-1.67	-0.993	-1.0372
0.6	-0.944	-1.2396	-1.6614	-0.9508	-0.9905
0.65	-0.8785	-1.1364	-1.7951	-0.8841	-0.9161
0.7	-0.892	-0.9786	-1.747	-0.7963	-0.8186
0.75	-0.6875	-0.7734	-1.5815	-0.6903	-0.7032
0.8	-0.568	-0.5416	-1.0482	-0.5693	-0.7745
0.85	-0.4365	-0.3271	-0.7878	-0.4367	-0.4365
0.9	-0.296	-0.1951	0.0901	-0.2955	-0.2922
0.95	-0.2195	-0.1444	0.5866	-0.1489	-0.1453
1	0	0	0	0	0

Table.2. Deflection of structural steel beam at mid span for mass 1.8kg at different values of α.

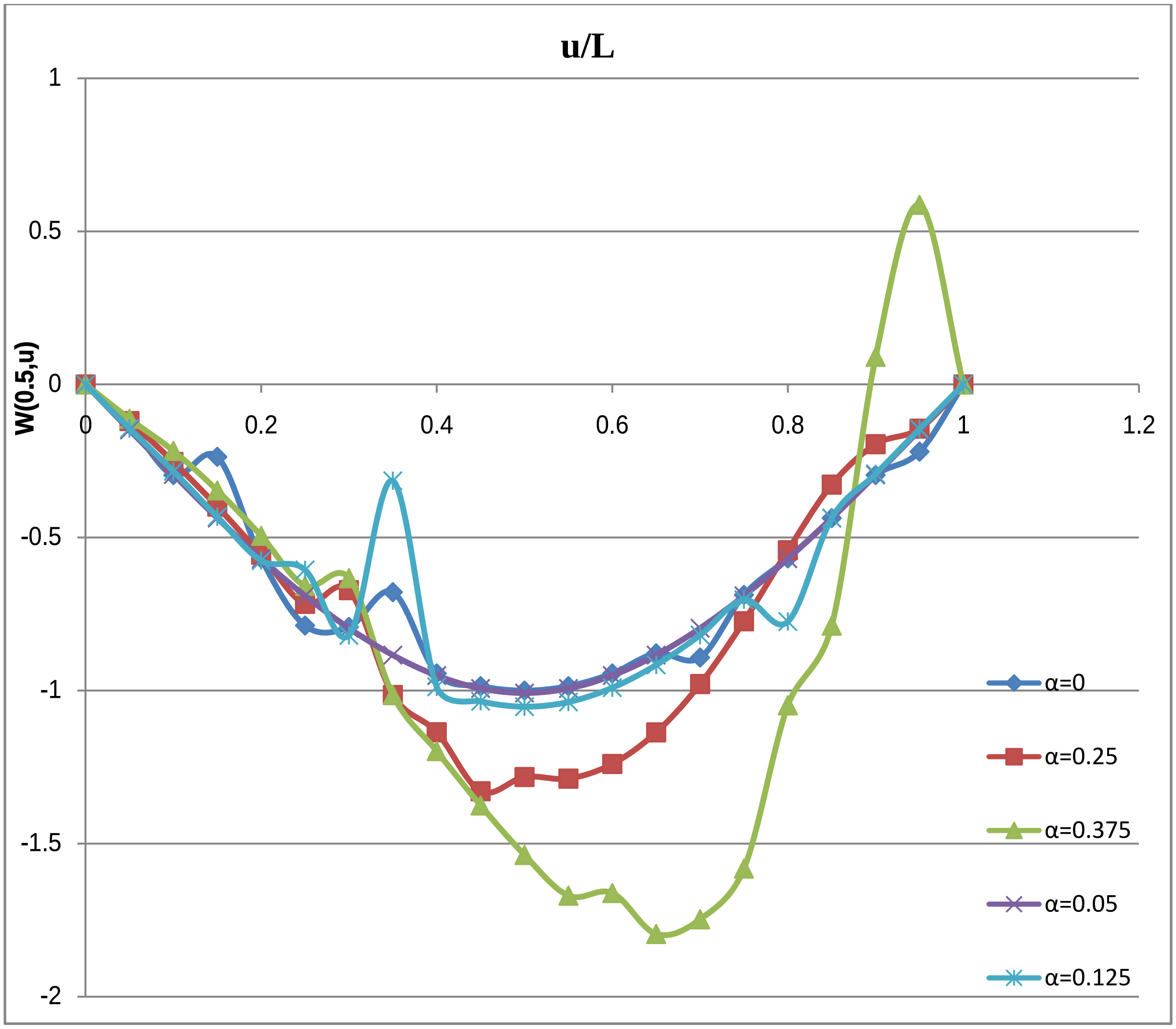


Fig.5.7 Mid-span deflection of beam traversed by a moving mass when $M/_{mL} = 0.6$

5.4. b. For Cantilever Beam (structural steel)

Beam is divided in to five stations with each interval of 0.2m.

For structural steel and mass of 0.9 kg

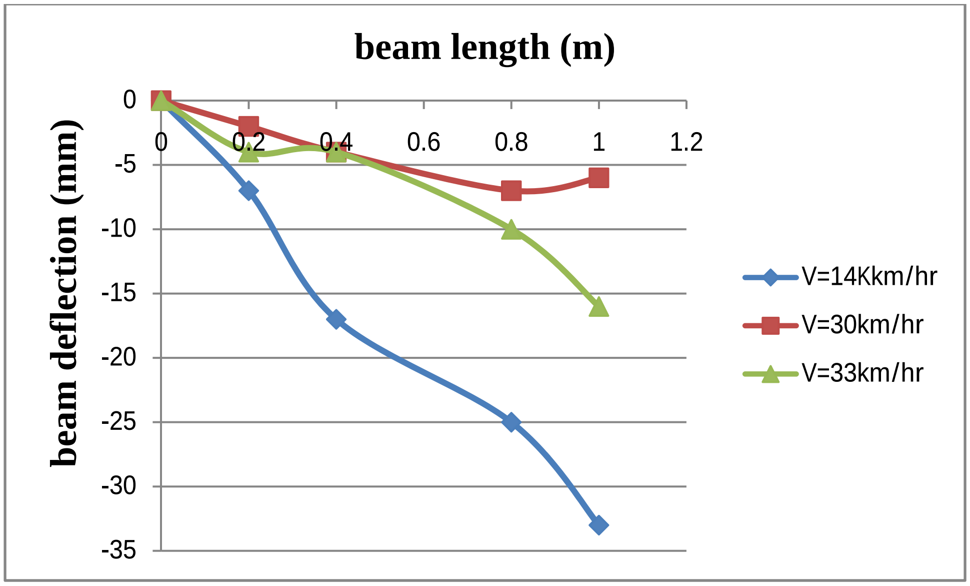


Fig.5.8 Deflection of cantilever beam at end point for different velocities.

For structural steel and mass of 1.8 kg

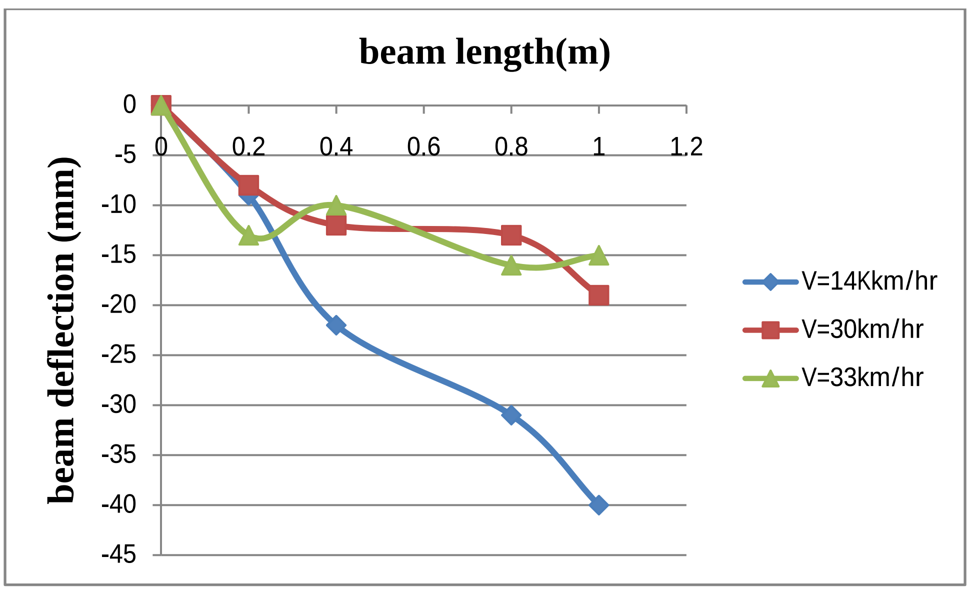


Fig.5.9 Deflection of cantilever beam at end point for different velocities for mass M=1.8kg

CHAPTER 6

-DISCUSSION-

6. DISCUSSIONS

Numerical results have been presented for the dynamic response of both simply supported beam and cantilever beam with a moving mass. Results have been shown for Deflection Vs Position of the beam. The following observations are made from the numerical Analysis:

1. In the case of simply supported beam, as the velocity of the moving mass increases maximum deflection of the beam also increases as well as the position of the maximum deflection deviates from the mid-point of the beam. As the weight of the moving mass increases, maximum deflection value also increases, this happens because of the inertia of the moving mass.
2. In case of Cantilever Beam, it is observed that as the velocity of the moving mass increases, the deflection at the free end point of the beam decreases. This is because at higher velocity of the moving mass, lower modes are not excited, which mainly contribute for larger dynamic deflection of the beam. As the lower modes of vibration contribute larger amplitude as compared to that of higher modes, dynamic deflection of the beam decreases.
3. It is also observed for a cantilever beam that as the mass of the moving body increases, the end deflection also increases. This is due to increase in the inertia of the moving mass.
4. The results obtained from the numerical analysis have been compared with the numerical results of a reference paper.

CHAPTER 7

- CONCLUSION AND SCOPE FOR FUTURE WORK-

7. CONCLUSION AND SCOPE FOR FUTURE WORK

An exact and direct modeling technique is presented in this work for modeling beam structures with various boundary conditions, subjected to a load moving at a constant speed. In order to validate the efficiency of the method presented, quantitative examples are given and results are compared with those available in the literature. In addition, the influence of a variation in the speed parameters of the system on the dynamic response of the beam was studied and the results were given in graphical form.

1. The objective of this work was to present a simple and direct technique to solve the problem of a beam traversed by a moving mass. The influences of variations of the travelling velocity and the effect of increase in the weight of the moving mass on the dynamic response are studied.

2. If the velocity of the load increases, the position of the maximum response on the beam occurs far from the midpoint. At very high speeds the maximum deflection of the beam occurs close to the end of the beam. For some values of the velocity the maximum response may occur before the middle of the beam. The dynamic response of the beam is more affected from the velocity of load than mass ratio of the system. It has been shown that**,** only the midpoint deflection or midpoint stresses in engineering calculations of the beam systems is insufficient. It brings out more accurate results to take into account the mass and velocity of the moving load and dynamic properties of carrying system in dynamic analysis.

3. The effect of the change of material on the dynamic response on both simply supported and cantilever beam is same.

SCOPE OF FUTURE WORK:

1. The present research can be extended to Timoshenko beam.
2. Acceleration of a travelling mass over a structural system, highly affects the dynamic response of the structural system. It can give engineers some advantages to make a more realistic modeling of structural systems under accelerating mass motion than classical methods that omit inertial effects of accelerating mass.
3. There are situations when a series of moving mass travels over a beam as a train travels over a bridge. Response of beams to such types of moving load is research worthy.

CHAPTER 8

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